Abington Heights School District
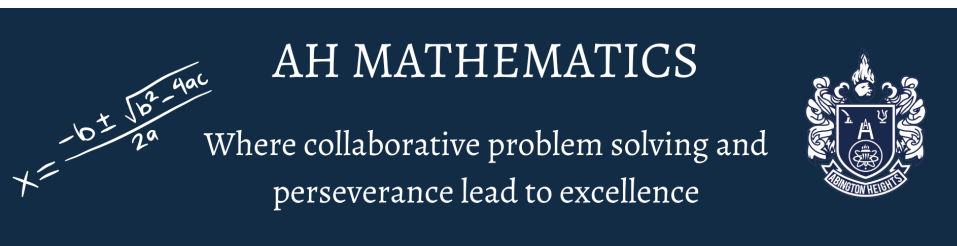# Precalculus AP Curriculum

In Precalculus AP, students develop their numeracy skills through the following areas of study:

1. Polynomial and Rational Functions
2. Exponential and Logarithmic Functions
3. Trigonometric and Polar Functions

Board Approval Date: June 7, 2023
Adoption: 2023 - 2024 SY
Review Date:

AH MATHEMATICS

Where collaborative problem solving and perseverance lead to excellence

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Abington Heights Math Framework

Stakeholders	Actions
Students	★ Engage in mathematical discussions, share their ideas openly, be inquisitive, seek to understand and learn more about mathematical concepts, and try their best daily. ★ Exhibit creativity and curiosity in problem solving individually and collaboratively. ★ Persevere in engaging and challenging daily mathematical practice. ★ Come prepared to learn every day.
Teachers	★ Create a safe and collaborative classroom environment where students feel vested in a shared vision for mathematical excellence. ★ Develop high quality instruction that meets the needs of all learners through differentiation. ★ Use a variety of 21st century methodologies to advance learning. ★ Partner with parents and guardians to support student success. ★ Establish a collaborative community within the building and amongst grade levels to ensure a cohesive level of instruction.

Building Leaders	★ Deeply understand the needs of teachers, students, the instructional materials being used, programs being implemented, and the expectations for state-level assessment scores
	  ○ Knowledgeable about program and grade level standards
	  ○ Ensure consistent and equal access to high-quality instructional materials and resources, building.
	★ Be partners with teachers, students and families:
	  ○ Provide guidance and support to the mathematical community.
	  ○ Understand needs of teachers, students and families.
	★ Trust the educators to make professional decisions based on program, student, and district needs.

Central Admin	★ Effectively communicate to the school board and community specific areas of need and how to support teachers and building leaders in a quest for mathematical excellence
	★ Deeply understand the needs of teachers, students, the instructional materials being used, programs being implemented, and the expectations for state-level assessment scores
	  ○ Have a common metric for mathematical excellence.
	  ○ Ensure consistent and equal access to high-quality instructional materials and resources, district.
	  ○ Re-examine best practices/curriculum routinely (6 years).
	★ Support a culture of collaboration between the other stakeholder groups to maintain the standard of excellence of the Abington Heights
	★ Trust the educators to make professional decisions based on program, student, and district needs.

Parents/ Community	★ Be a strong support system and contribute by building a positive math community for students. ★ Encourage a positive math mindset. ★ Have conversations with their children about school and ask what they are learning about in school. ★ Be open, receptive to the district's ideas about student learning and reach out to teachers/school to learn more about how they can support. ★ Trust the educators to make professional decisions based on program, student, and district needs.

School Board	★ Provide the fiscal resources to support:
	  ○ Highly qualified professionals for mathematics
	  ○ High-quality instructional materials
	  ○ Effective and efficient math interventions for remediation
	  ○ Professional development for math content and instructional practices
	★ Trust the educators to make professional decisions based on program, student, and district needs.

Precalculus AP Scope and Sequence

Month	Unit	Estimated Number of Weeks
September	Polynomial and Rational Functions	4
October	Polynomial and Rational Functions	4
November	Exponential and Logarithmic Functions	4
December	Exponential and Logarithmic Functions	3
January	Exponential and Logarithmic Functions	3
February	Trigonometric and Polar Functions	4
March	Trigonometric and Polar Functions	4
April	Trigonometric and Polar Functions	2
	AP Exam Review	2
May	AP Exam Review	1
	Limits	3
June	Final Exam Review	1

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 1 Polynomial and Rational Functions	What are functions and how can functions be built and represented (graphically, numerically, and algebraically)? How can I use graphs of equations in solving real-life problems? How can I describe the characteristics of and recognize graphs of the parent functions? What is a polynomial? How does the degree of a polynomial affect its graph? How do the factors of a polynomial affect its graph?	Change in Tandem Rates of Change Rates of Change in Linear and Quadratic Functions Polynomial Functions and Rates of Change Polynomial Functions and Complex Zeros Polynomial Functions and End Behavior Rational Functions and End Behavior Rational Functions and Zeros Rational Functions and Vertical Asymptotes Rational Functions and Holes	Describe how the input and output values of a function vary together by comparing function values. Construct a graph representing two quantities that vary with respect to each other in a contextual scenario. Compare the rates of change at two points using average rates of change near the points. Describe how two quantities vary together at different points and over different intervals of a function.	CC.2.1.HS.F.3 CC.2.1.HS.F.4 CC.2.1.HS.F.6 CC.2.1.HS.F.7 CC.2.2.HS.D.1 CC.2.2.HS.D.2 CC.2.2.HS.D.3 CC.2.2.HS.D.4 CC.2.2.HS.D.5 CC.2.2.HS.D.6 CC.2.2.HS.D.7 CC.2.2.HS.D.8 CC.2.2.HS.D.9 CC.2.2.HS.D.10 CC.2.2.HS.C.1 CC.2.2.HS.C.2 CC.2.2.HS.C.3 CC.2.2.HS.C.4	Flipcharts Desmos Activities Calc-Medic Activities Delta Math WeBWorK	Desmos Activities Delta Math WeBWorK Homework Quizzes Test(s)

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 1 Polynomial and Rational Functions (continued)	What techniques and methods can be used to find the roots of higher degree polynomial functions? How can I use the Fundamental Theorem of Algebra to determine the number of zeros of polynomial functions? How can I use the domain and asymptotes of a rational function to find and analyze its graph?	Equivalent Representations of Polynomial and Rational Expressions Transformations of Functions Function Model Selection and Assumption Articulation Function Model Construction and Application	Determine the average rates of change for sequences and functions, including linear, and other function types. Determine the change in the average rates of change for linear, quadratic, and other function types. Identify key characteristics of polynomial functions related to rates of change. Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.	CC.2.2.HS.C.5 CC.2.2.HS.C.6		

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 1 Polynomial and Rational Functions (continued)			Determine if a polynomial function is even or odd. Describe end behaviors of polynomial functions. Describe end behaviors of rational functions. Determine the zeros of rational functions. Determine vertical asymptotes of graphs of rational functions. Determine holes in graphs of rational functions. Rewrite polynomial and rational expressions in equivalent forms.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 1 Polynomial and Rational Functions (continued)			Determine the quotient of two polynomial functions using long division. Rewrite the repeated product of binomials using the binomial theorem. Construct a function that is an additive and/or multiplicative transformation of another function. Identify an appropriate function type to construct a function model for a given scenario. Describe assumptions and restrictions related to building a function model.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 1 Polynomial and Rational Functions (continued)			Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model. Construct a rational function model based on a context.			
Unit 2 Exponential and Logarithmic Functions	What is a logarithm? How are logarithms and exponentials related? How can I graph exponential functions and logarithmic functions? Where do I use logarithmic functions to model and solve real-life problems?	Change in Arithmetic and Geometric Sequences Change in Linear and Exponential Functions Exponential Functions Exponential Function Manipulation Exponential Function Context and Data Modeling	Express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers. Express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers. Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.	CC.2.1.HS.F.1 CC.2.1.HS.F.3 CC.2.1.HS.F.4 CC.2.2.HS.D.5 CC.2.2.HS.D.8 CC.2.2.HS.D.9 CC.2.2.HS.D.10 CC.2.2.HS.C.2 CC.2.2.HS.C.3 CC.2.2.HS.C.4 CC.2.2.HS.C.5 CC.2.2.HS.C.6	Flipcharts Desmos Activities Calc-Medic Activities Delta Math WeBWorK	Desmos Activities Delta Math WeBWorK Homework Quizzes Test(s)

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 2 Exponential and Logarithmic Functions (continued)	How can I use properties of logarithms to expand, condense, rewrite, or evaluate logarithmic expressions? How do I use exponential growth functions to model and solve real-life problems?	Competing Function Model Validation Composition of Functions Inverse Functions Logarithmic Expressions Inverses of Exponential Functions Logarithmic Functions Logarithmic Function Manipulation Exponential and Logarithmic Equations and Inequalities Logarithmic Function Context and Data Modeling Semi-Log Plots	Describe similarities and differences between linear and exponential functions. Identify key characteristics of exponential functions. Rewrite exponential expressions in equivalent forms. Construct a model for situations involving proportional output values over equal-length input-value intervals. Apply exponential models to answer questions about a data set or contextual scenario.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 2 Exponential and Logarithmic Functions (continued)			Construct linear, quadratic, and exponential models based on a data set. Validate a model constructed from a data set. Evaluate the composition of two or more functions for given values. Construct a representation of the composition of two or more functions. Rewrite a given function as a composition of two or more functions. Determine the input-output pairs of the inverse of a function. Determine the inverse of a function on an invertible domain.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 2 Exponential and Logarithmic Functions (continued)			Evaluate Logarithmic expressions. Construct representations of the inverse of an exponential function with an initial value of 1. Identify key characteristics of logarithmic functions. Rewrite logarithmic expressions in equivalent forms. Solve exponential and logarithmic equations and inequalities. Construct the inverse function for exponential and logarithmic functions. Construct a logarithmic function model.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 2 Exponential and Logarithmic Functions (continued)			Determine if an exponential model is appropriate by examining a semi-log plot of a data set. Construct the linearization of exponential data.			
Unit 3 Trigonometric and Polar Functions	How do trigonometric and circular functions model real-world problems and their solutions? What are the six trigonometric ratios and how do they relate to each other? What do the key features of a trigonometric function represent? How do trigonometric and circular functions model real-world problems and their solutions?	Periodic Phenomena Sine, Cosine, and Tangent Sine and Cosine Function Values Sine and Cosine Function Graphs Sinusoidal Functions Sinusoidal Function Transformations Sinusoidal Function Context and Data Modeling	Construct graphs of periodic relationships based on verbal representations. Describe key characteristics of a periodic function based on a verbal representation. Determine the sine, cosine, and tangent of an angle using the unit circle. Determine coordinates of points on a circle centered at the origin.	CC.2.2.HS.D.2 CC.2.2.HS.D.8 CC.2.2.HS.D.9 CC.2.2.HS.D.10 CC.2.2.HS.C.2 CC.2.2.HS.C.3 CC.2.2.HS.C.4 CC.2.2.HS.C.6 CC.2.2.HS.C.7 CC.2.2.HS.C.8 CC.2.2.HS.C.9 CC.2.3.HS.A.7 CC.2.3.HS.A.9	Flipcharts Desmos Activities Calc-Medic Activities Delta Math WeBWorK	Desmos Activities Delta Math WeBWorK Homework Quizzes Test(s)

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 3 Trigonometric and Polar Functions (continued)	How can I use identities to simplify trigonometric expressions and prove other identities? How can I use identities to solve trigonometric equations? How can I use the Law of Sines and the Law of Cosines to solve real-world problems? What are the similarities and differences between the polar and Cartesian coordinate systems?	The Tangent Function Inverse Trigonometric Functions Trigonometric Equations and Inequalities The Secant, Cosecant, and Cotangent Functions Equivalent Representations of Trigonometric Functions Trigonometric and Polar Coordinates Polar Function Graphs Rates of Change in Polar Functions	Construct representations of the sine and cosine functions using the unit circle. Identify key characteristics of the sine and cosine functions. Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function. Construct sinusoidal function models of periodic phenomena. Construct representations of the tangent function using the unit circle. Describe key characteristics of the tangent function.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 3 Trigonometric and Polar Functions (continued)			Describe additive and multiplicative transformations involving the tangent function. Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain. Solve equations and inequalities involving trigonometric functions. Identify key characteristics of functions that involve quotients of the sine and cosine functions.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 3 Trigonometric and Polar Functions (continued)			Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity. Rewrite trigonometric expression in equivalent forms with sine and cosine sum Identities. Solve equations using equivalent analytic representations of trigonometric functions. Determine the location of a point in the plane using both rectangular and polar coordinates. Construct graphs of polar functions.			

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 3 Trigonometric and Polar Functions (continued)			Describe characteristics of the graph of a polar function.			
Unit 4 Limits	What is a limit? How can a limit be used to determine the continuity of a function? How are limits connected to Calculus?	Introduction to Limits Techniques for Evaluating Limits The Tangent Line Problem Limits at Infinity	Use the definition of a limit to estimate limits Determine whether limits of a function exist Use properties and operations of limits to find limits Find limits of polynomial and rational functions by direct substitution Use the dividing out and rationalizing techniques to find limits of functions Approximate limits of functions graphically and numerically		Limits flipchart Desmos Activities Calc-Medic Activities Delta Math WeBWorK	Desmos Activities Delta Math WeBWorK Homework Quiz

Unit	Essential Questions	Content	Skills	PA Core Standards	Activities	Assessment/ Evidence of Learning
Unit 4 Limits (continued)			Evaluate one-sided limits of functions Evaluate limits of difference quotients Use a tangent line to approximate the slope of a graph at a point Use the limit definition of slope to find the exact slopes of graphs Find derivatives of functions and use derivatives to find slopes of graphs			

Portrait of an Abington Heights Mathematician

By the end of Precalculus, students will:

Functions and Their Graphs	Polynomial and Rational Functions	Exponential and Logarithmic Functions	Trigonometric Functions	Trigonometric Applications
☐ Graph and analyze functions and use their properties to make connections between the different representations ☐ Analyze functions and graphs of functions, including characteristics such as increasing/decreasing, odd/even, relative and absolute minima and maxima ☐ Recognize graphs of common functions and use rigid and nonrigid transformations ☐ Combine and compose functions ☐ Find and graph inverse functions	☐ Model real-life problems using quadratic functions ☐ Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs ☐ Analyze and sketch graphs of rational functions, including domain, range, asymptotes, and discontinuities	☐ Recognize, evaluate, and graph exponential functions ☐ Use exponential functions to model and solve real-life problems ☐ Recognize, evaluate, and graph exponential functions ☐ Use logarithmic functions to model and solve real-life problems ☐ Solve exponential and logarithmic equations	☐ Apply radian measure of an angle and the unit circle to analyze the trigonometric functions ☐ Extend the concept of similarity to determine arc lengths and areas of sectors of circles ☐ Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs ☐ Prove the Pythagorean identity and use it to calculate trigonometric ratios ☐ Apply trigonometric ratios to solve problems involving right triangles ☐ Solve trigonometric equations, including using algebraic techniques, Pythagorean identities, and multiple angles	☐ Use fundamental trigonometric identities to evaluate trigonometric functions, and to simplify and rewrite trigonometric expressions ☐ Use the Law of Sines and/or the Law of Cosines to solve triangles ☐ Solve real-life problems using the Law of Sines and/or the Law of Cosines ☐ Find the area of oblique triangles